Variation with cavity Q of the beat frequency between axial modes of a gas laser

1968 J. Phys. A: Gen. Phys. 2 87

(http://iopscience.iop.org/0022-3689/2/1/012)

Variation with cavity Q of the beat frequency between axial modes of a gas laser

L. ALLEN, D. G. C. JONES and M. D. SAYERS
University of Sussex, School of Mathematical and Physical Sciences, Brighton, Sussex
MS. received 4th June 1968

Abstract. Investigations at 0·633 μm and 1·15 μm in the He–Ne laser show that beat frequencies at about $c/2l$ and $2c/2l$ between neighbouring axial modes are found to shift linearly with inserted cavity loss. These effects are explained using Lamb theory. Information is gained concerning the stability of mode distributions. The effect is also investigated in the argon ion laser at 0·488 μm.

1. Introduction

Investigation of the beat notes between modes in gas lasers was carried out in the first gas laser (Javan *et al.* 1961) and the frequency interval between beats confirmed the existence of the axial and transverse modes predicted by Fox and Li (1961). Bennett (1962) extended the work and analysed the splitting of the beats at about $c/2l$ in terms of the 'pulling' and 'pushing' processes which occur when two or more axial modes interact with an inhomogeneously broadened line. This analysis was confirmed and extended by the semiclassical laser theory of Lamb (1964). Fork and Pollock (1965) computed the mode intensities and beat frequencies for the case of two axial modes as a function of resonator tuning using Lamb theory, but not for the case of variable $\bar{N}$ and Q. McFarlane (1964) made measurements of the variation of beat frequency between two axial modes of a laser as a function of detuning. His measurements were compared with Lamb theory but the mode interaction terms were ignored.

In the present work the frequency variation between consecutive pairs of axial modes in a gas laser has been investigated as a function of cavity Q. The beat frequency is found to change with Q and this effect, first noticed in the 0·633 μm laser line of He–Ne, has been further investigated at 1·15 μm and on the 0·488 μm line in an Ar^+ laser. The effect is described theoretically by the rigorous application of Lamb theory for three modes, with all interaction terms taken into account.

2. Experimental details

The Q of a gas laser cavity may be slowly varied by a measurable amount using a skew Brewster angle flat (Allen *et al.* 1968). Unlike the technique of detuning the flat from the Brewster angle about an axis perpendicular to the direction of propagation of the laser light (Witteman 1966, Schleusener and Read 1966, Kiss and Salamon 1967), the skew Brewster angle flat is detuned by rotation about an axis parallel to the direction of propagation. This has the effect of removing many of the undesirable features of the earlier device and of allowing a more accurately determinable variation of the reflection losses and cavity Q.

The laser light was detected by a photomultiplier, and the mode beat frequencies at about $c/2l$ and $2c/2l$ were observed in the following way (figure 1). Part of the signal from the photomultiplier was mixed with that of a local r.f. oscillator whose frequency could be swept about a pre-set mean value in synchronization with the output of an oscilloscope time base. The difference frequency from this mixer, nominally 14 MHz, was fed to a communications receiver and its i.f. output fed to the y_1 plates of the oscilloscope. The second part of the signal was mixed with the output of a frequency doubler driven by the same local r.f. oscillator. The 28 MHz difference frequency was fed to a second communications receiver and its i.f. output fed to the y_2 plates of the oscilloscope. The frequency spectrum of the signal over two small ranges of frequency near $c/2l$ and $2c/2l$

was thus displayed on the two beams of the oscilloscope in the manner of a conventional r.f. spectrum analyser. The range displayed could be varied between 0 and 50 kHz cm^{-1} by choice of the sweep range of the local oscillator. In this way, a 50 kHz shift in beat frequency could be made to correspond to a shift of about 10 cm on the oscilloscope

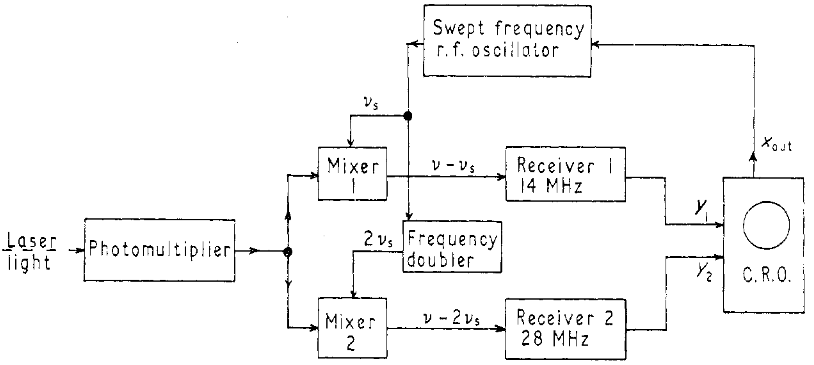

Figure 1. Block diagram of beat detection system.

screen; this was equivalent typically to a change in cavity loss of about 1% for the 0·633 μm line. The resolution of the system was effectively determined by the bandwidths of the crystal filters in the i.f. stages of the communications receivers; the bandwidth of the filters combined with a careful choice of sweep enabled a resolution of about 500 Hz to be achieved for the beat notes.

The beat frequencies were examined for a large number of inserted losses and the resultant frequency shift plotted against the change in loss. Such shifts have been observed for various types of cavity but the most reliable results occurred when a plano-concave cavity was used. This was because the skew Brewster angle plate could then be placed near the plane mirror where the laser beam was narrow and essentially non-divergent, and the loss due to the plate could be accurately determined. If the beam were wide, the inserted loss would become excessively dependent upon the optical homogeneity of the glass slide used. The experiment was performed for a number of cavity lengths and for different dispositions of the discharge tube in the cavity.

The beat at around $c/2l$ from the argon laser line had an envelope about 30 kHz wide within which there were a large number of components whose positions varied with time. This structure was presumably due to instabilities in the plasma. The appearance of the beat remained essentially the same over a large range of inserted loss except that its mean frequency shifted. In the case of the He–Ne laser lines there were a number of narrow beats whose width ranged between 0·5 and 4 kHz. The frequency of each beat was found to shift linearly with loss (see figure 2). An exception to this behaviour occurred when the

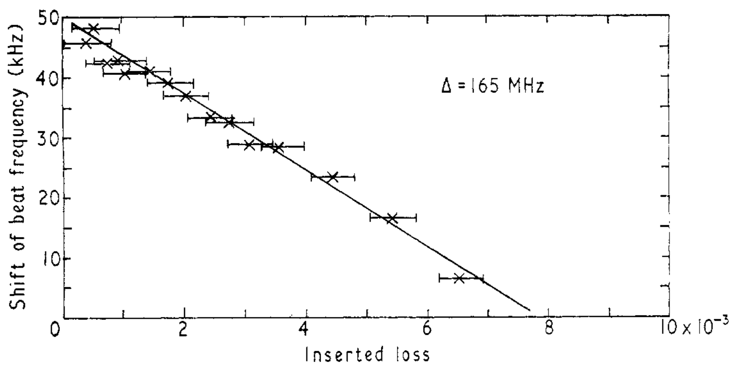

Figure 2. Shift of beat frequency against inserted loss for the 0·633 μm line.

pattern simplified to one sharply defined beat indicating that the modes had become locked. This was confirmed by the fact that the beat at about $2c/2l$ also became well defined with a measured frequency of twice that of the single beat at around $c/2l$ to within the limits of experimental measurement (± 500 Hz).

For a given loss the beat distribution stayed steady for a few minutes and then a new distribution appeared, the change being caused apparently by some form of cavity perturbation. This explains why a set of results of beat frequency against loss recorded over a time longer than a few minutes did not yield a single line, but a number of straight lines of approximately the same slope with different intercepts.

It should, perhaps, be noted that no special attempts were made to stabilize the length of the cavity. However, the laser was shielded from thermal currents and dust, and no measurements were made until the system had been running for some hours. All measurements were made with the laser in (0, 0) modes.

3. Theory

Lamb (1964) has derived equations which determine the intensities of three interacting axial modes. If it is assumed that there is no phase correlation between the modes, then for a steady state these equations form a linear system. It is convenient to express them in matrix form as

$$\boldsymbol{\alpha} = \hat{\mathbf{S}} W \tag{1}$$

where

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}, \qquad W = \begin{pmatrix} E_1^2 \\ E_2^2 \\ E_3^2 \end{pmatrix} \qquad \text{and } \hat{\mathbf{S}} = \begin{pmatrix} \beta_1 & \theta_{12} & \theta_{13} \\ \theta_{21} & \beta_2 & \theta_{23} \\ \theta_{31} & \theta_{32} & \beta_3 \end{pmatrix}.$$

$\boldsymbol{\alpha}$ represents the gain parameters for the three modes, $\boldsymbol{W}$ describes their steady-state intensities, and $\hat{\mathbf{S}}$ is a matrix whose diagonal elements represent saturation parameters and other elements represent interaction terms. All symbols, unless otherwise stated, have the same meaning as in Lamb's (1964) paper. If differences are taken between the mode frequencies given by Lamb (equation 114) and arranged so that when $\nu_3 > \nu_2 > \nu_1$ all beat frequencies are positive, then

$$\boldsymbol{\nu} = \boldsymbol{\Omega} + \boldsymbol{\sigma} + \hat{\mathbf{T}} W \tag{2}$$

where

$$\boldsymbol{\nu} = \begin{pmatrix} \nu_2 - \nu_1 \\ \nu_3 - \nu_2 \\ \nu_3 - \nu_1 \end{pmatrix}, \qquad \boldsymbol{\Omega} = \begin{pmatrix} \Omega_2 - \Omega_1 \\ \Omega_3 - \Omega_2 \\ \Omega_3 - \Omega_1 \end{pmatrix}, \qquad \boldsymbol{\sigma} = \begin{pmatrix} \sigma_2 - \sigma_1 \\ \sigma_3 - \sigma_2 \\ \sigma_3 - \sigma_1 \end{pmatrix}$$

and

$$\hat{\mathbf{T}} = \begin{pmatrix} \tau_{21} - \rho_1 & \rho_2 - \tau_{12} & \tau_{23} - \tau_{13} \\ \tau_{31} - \tau_{21} & \tau_{32} - \rho_2 & \rho_3 - \tau_{23} \\ \tau_{31} - \rho_1 & \tau_{32} - \tau_{12} & \rho_3 - \tau_{13} \end{pmatrix}.$$

$\boldsymbol{\Omega}$ describes the differences in frequency between the passive cavity resonances, $\boldsymbol{\sigma}$ describes 'pulling' of the modes and $\hat{\mathbf{T}}$ describes intensity-dependent 'pushing' and interaction terms. Solving (1) for the intensities W and substituting in (2) gives

$$\boldsymbol{\nu} = \boldsymbol{\Omega} + \boldsymbol{\sigma} + \hat{\mathbf{T}} \hat{\mathbf{S}}^{-1} \boldsymbol{\alpha}. \tag{3}$$

$\boldsymbol{\alpha}$ consists of two terms, of which only the first includes the cavity loss. Assuming the cavity to have the same Q at the three mode frequencies, it is possible to write $\boldsymbol{\alpha} = k_1 L\boldsymbol{I} + \boldsymbol{F}$, where

$$k_1 = \frac{\nu}{2LQ} = \frac{\Delta}{2\pi}, \qquad \boldsymbol{F} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$$

L is the cavity loss and Δ the passive cavity mode separation, and where

$$f_n = \frac{1}{2}\frac{\nu \bar{N}\mathscr{P}^2}{\epsilon_0 \hbar K u} Z_i(\nu_n - \omega).$$

Substituting this expression for $\boldsymbol{\alpha}$ into (3) gives

$$\boldsymbol{\nu} = \boldsymbol{\Omega} + \boldsymbol{\sigma} + k_1 L \hat{\mathbf{T}}\hat{\mathbf{S}}^{-1} I + \hat{\mathbf{T}}\hat{\mathbf{S}}^{-1} F. \tag{4}$$

It can be seen from this equation that the column vector representing the beat frequencies is a linear function of the cavity loss L. It remains to show that this linear dependence is not masked in practice by changes in $\boldsymbol{\sigma}$, $\hat{\mathbf{T}}$, $\hat{\mathbf{S}}$ or F, which are all functions of the detuning of the modes from the line centre. Equation (4) also holds for the case of two interacting modes but the matrices then reduce to a particularly simple form:

$$\hat{\mathbf{S}} = \begin{pmatrix} \beta_1 & \theta_{12} \\ \theta_{21} & \beta_2 \end{pmatrix}, \qquad \hat{\mathbf{T}} = (\tau_{21} - \rho_1 \quad \rho_2 - \tau_{12})$$

$$\boldsymbol{\Omega} = (\Omega_2 - \Omega_1), \qquad F = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}, \qquad \boldsymbol{\sigma} = (\sigma_2 - \sigma_1).$$

The components of the matrices $\hat{\mathbf{S}}$ and $\hat{\mathbf{T}}$ are as defined in Lamb's paper (equations 109–129) with the approximation that the pulled mode frequency ν is replaced by the cavity frequency Ω, wherever ν appears in an argument of a Lorentzian or Gaussian function. If all frequencies are written in terms of the axial mode spacing Δ, the components of $\hat{\mathbf{S}}$ and $\hat{\mathbf{T}}$ become functions of single detuning parameter a, where the detuning of the central mode from the line centre is given by $\nu_2 - \omega = a\Delta$. The analysis is now concerned with evaluating the beat frequencies as functions of loss and detuning for various values of γ, Ku and N_2.

It should be noted that not all possible detunings lead to stable three-mode (or even two-mode) solutions and it has therefore also been necessary to determine the intensities E_1^2, E_2^2, E_3^2 in the steady state for each value of a and of loss. If the solution predicts a negative intensity for one or more modes it is clear that this particular distribution of modes is not allowable. This analysis is not concerned with the evolution of the system into a steady state but only with the properties of those final states which can actually exist.

It is possible to avoid the need to know the value of $\bar{N}$ by expressing the two parts of α_n in terms of the inserted loss and the value of the inserted loss necessary to extinguish the laser. From Lamb (equation 82)

$$\alpha_n = -\frac{\nu}{2Q_n} + \frac{1}{2}\frac{\nu \bar{N}\mathscr{P}^2}{\epsilon_0 \hbar K u} Z_i(\nu_n - \omega).$$

Expanding

$$Z_i(\nu_n - \omega) \simeq \frac{-2\gamma_{ab}}{Ku} + \sqrt{\pi}\exp\left\{-\left(\frac{\nu_n - \omega}{k_n}\right)^2\right\}$$

and writing

$$\frac{1}{2}\frac{\nu \bar{N}\mathscr{P}^2}{\epsilon_0 \hbar K u} = k, \qquad \frac{\nu}{2Q_n} = k_1 L$$

as before, we have

$$\alpha_n = -\left(k_1 L + \frac{2\gamma_{ab}}{Ku} k\right) + \sqrt{\pi} k \exp\left\{-\frac{(\nu_n - \omega)^2}{Ku}\right\}.$$

If L_{max} is the loss which must be inserted to extinguish the laser and L_c is the cavity loss, then at threshold $\alpha_n = 0$ and

$$k = k_1 L_0 \left[\sqrt{\pi}\exp\left\{-\left(\frac{\nu_n - \omega}{Ku}\right)^2\right\} - \frac{2\gamma_{ab}}{Ku}\right]^{-1}$$

where $L_0 = L_{max} + L_c$.

Since the first mode to begin oscillation must be very near to the line centre, and since $\gamma_{ab} \ll Ku$,

$$k \simeq \frac{k_1}{\sqrt{\pi}} L_0.$$

The maximum inserted loss required to extinguish the laser is measurable and L_c can be calculated. The ratio

$$\frac{k}{k_1} = \frac{L_{max} + L_c}{\sqrt{\pi}}$$

is therefore known. In equation (4) a component α_i of $\boldsymbol{\alpha}$ can therefore now be written as

$$\alpha_i = \frac{\Delta}{2\pi}\left[-\left(L + \frac{2\gamma_{ab}}{Ku}\frac{L_0}{\sqrt{\pi}}\right) + L_0 \exp\left\{-\left(\frac{a-2+i}{d}\right)\right\}\right]$$

where $d = Ku/\Delta$. Similarly, expanding the function $Z_r(\nu_n - \omega)$ it is possible to show that

$$\sigma_i = \frac{\Delta}{2\pi} 2L_0 \left[\left(\frac{a-2+i}{d}\right)\frac{\gamma_{ab}}{Ku}\exp\left(\frac{\gamma_{ab}}{Ku}\right)^2 \exp\left\{-\left(\frac{a-2+i}{d}\right)^2\right\} - 2\left(\frac{a-2+i}{d}\right)\right]$$

and hence the components of $\boldsymbol{\sigma}$ can be computed. Substituting these values of $\boldsymbol{\alpha}$ and $\boldsymbol{\sigma}$ into equation (3) allows the value of the beat frequency to be evaluated for a wide range of values of the detuning parameter a.

4. Predictions of the theory

The linear three-mode theory gives solutions for the mode intensities for all detunings except those which infer a symmetric distribution of modes about the line centre, i.e. $a = 0$, $0{\cdot}5\Delta$, Δ. In these cases, there is strong competition and it is expected that one of the modes would cease to oscillate, leaving a simple two-mode system. The width of the region about the symmetric positions in which this competition is strong enough to extinguish one of the modes is dependent on the size of γ_{ab}, being less than $0{\cdot}1\Delta$ when γ_{ab} is less than $0{\cdot}5\Delta$. This means that stable solutions can be obtained in this case for detunings where $0{\cdot}1\Delta \leqslant a \leqslant 0{\cdot}4\Delta$.

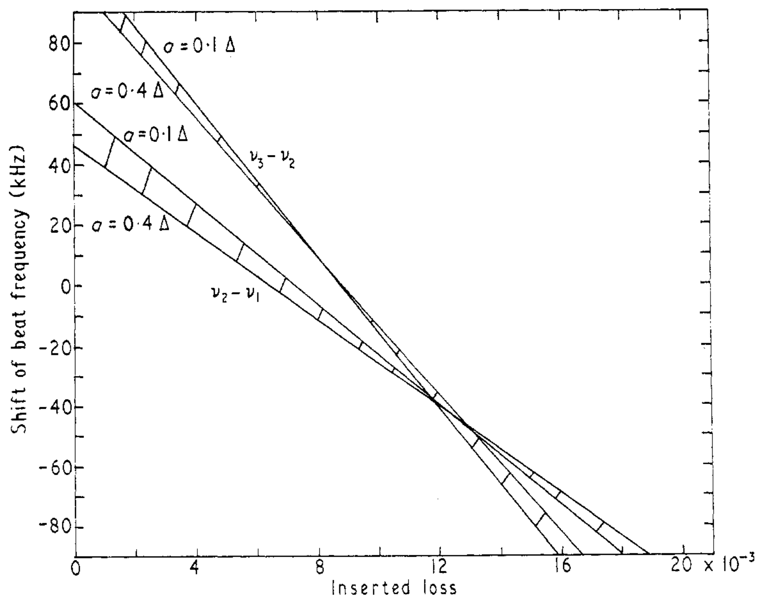

Figure 3. Theoretical prediction of shift of beat frequency against inserted loss for the He–Ne $0{\cdot}633\ \mu$m line: $Ku = 6\Delta$, $\gamma_{ab} = 0{\cdot}3\Delta$, $L_0 = 0{\cdot}03$, $L_c = 0{\cdot}01$ and $\Delta = 130$ MHz.

The factor $\Delta/2\pi$ appears in all the terms on the right-hand side of equation (3) giving a linear dependence of the beat frequency with the inverse of the cavity length. The effects of changes in $N_2/\bar{N}$ and $N_4/\bar{N}$ on the beat frequency are less than 1%, so that the position of the active medium in the cavity should not be important.

As a specific case, the results for the 6·33 μm He–Ne laser may be considered in greater detail. A value of $Ku = 6\Delta$ has been assumed, corresponding to a Doppler width $\Delta\nu_D = 1400$ MHz (Allen *et al.* 1967). L_0 is taken to be 0·03. A plot of the calculated beat frequency shift against inserted loss is shown in figure 3 for a value of $\gamma_{ab} = 0{\cdot}3\Delta$. The 'fans' show the limits of $\nu_2-\nu_1$ and $\nu_3-\nu_2$ for detuning of the modes given by $0{\cdot}1\Delta < a < 0{\cdot}5\Delta$. If the detuning of the modes from the line centre remains constant while the loss is changed, the beat frequency shift plotted against loss is represented by a single straight line. If a perturbation causes a sudden change in detuning, then the effect is to produce a straight line of similar slope to the original, but laterally displaced from it. If the detuning is continually changing, the beat frequency should follow a curve lying within the theoretical 'fan'. It can be seen from the figure that the separation of the $\nu_2-\nu_1$ and $\nu_3-\nu_2$ beats is dependent on the loss, being about 5 kHz near threshold and 30 kHz at maximum intensity.

For the $\nu_2-\nu_1$ beat, the slope within the 'fan', $\partial(\nu_2-\nu_1)/\partial L$, is about $(-8\pm1)\times10^6$ s^{-1} for $\Delta = 1{\cdot}3\times10^8$ s^{-1}. This slope is a function of γ_{ab}, but the variation is less than 25% for $0{\cdot}1\Delta \leqslant \gamma_{ab} \leqslant 0{\cdot}4\Delta$ and, since the variation with detuning (for fixed γ_{ab}) is of the order of 15%, it reduces the sensitivity of the experiment as a means of determining γ_{ab}.

5. Comparison of theory with experiment

The experimental results fall into three groups, those concerned with the 0·633 μm line, those with the 1·15 μm line and those with the 0·488 μm line. The results exhibit very similar over-all properties but, since the width of the 0·488 μm beat note was considerably larger than any other, the measurements of its displacement were much less precise.

For all experimental conditions, graphs of beat frequency against inserted loss were straight lines of negative slope. However, the measured gradients of the straight lines often varied from one experiment to the next, even for identical experimental arrangements. Typical results for the three laser lines are shown in figures 2 and 4.

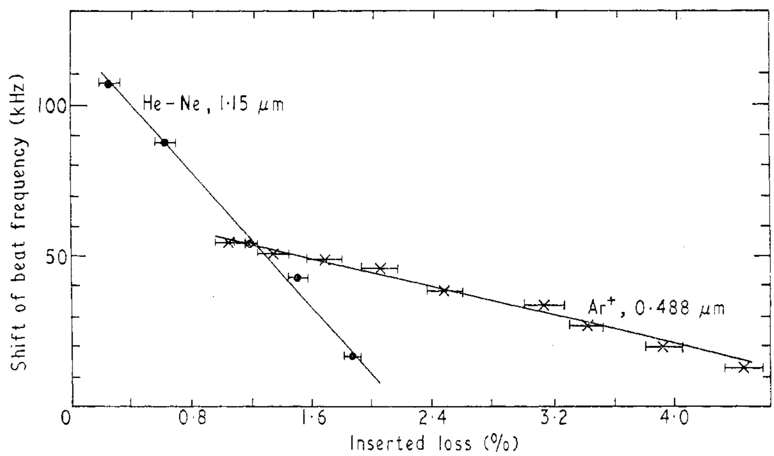

Figure 4. Shift of beat frequency against inserted loss for the 1·15 μm He–Ne laser line and for the 0·488 μm Ar$^+$ line.

Theory predicts that a plot of beat frequency against inserted loss should be a straight line, but that its gradient depends on the positions of the interacting modes with respect to the centre of the Doppler curve. The fact that straight lines are obtained experimentally seems to suggest that the configuration of the modes may be relatively stable for periods of the order of minutes. Discontinuities in the straight lines are apparently the result of some external perturbation of the cavity, such as a current of air or a dust particle, changing the mode distribution from one stable configuration to another. This explanation is in agreement with the results of the three-mode theory where it has been shown that a slight movement of the interacting modes could cause the radiation field to change from a stable three-mode, to an unstable three-mode, and hence to a stable two-mode configuration. Presumably, if the interaction of four or more modes were considered such changes would be more likely, as the regions for stable multimode oscillation would be smaller.

There are two sets of parameters which may be distinguished in the quantitative comparison of theory and experiment, those describing the length of the cavity and the disposition of the discharge tube in it, and those which define the atomic properties of the gas. It was not possible to vary the cavity length sufficiently to observe the anticipated dependence of beat frequency on length. To obtain a variation in the gradient large compared with the random variations observed, the cavity length would have had to be changed by a large factor. This was not possible since for a long cavity the axial mode spacing is so small that it is difficult to restrict oscillation to a few modes; conversely, for a short cavity the beat frequency is too high to be detected by conventional photomultipliers. As predicted by the theory, the variation of the parameters $\bar{N}/N_2$ and $\bar{N}/N_4$ had little effect upon the observed gradients, and so the ratio of cavity length to the length of excited gas was unimportant.

The important atomic parameter to be considered is γ_{ab}. The results for the three laser lines can most easily be discussed separately. There are a number of quoted values of γ_{ab} for the He–Ne 1·15 μm line (Bolwijn 1965, 1966, 1967, Bennett *et al.* 1965). A figure between 40 and 70 MHz appears most probable although one result considerably in excess of this has been quoted (Schweitzer *et al.* 1967). The gradients of the theoretical curves are not strongly dependent on γ_{ab}. The results for 1·15 μm suggest a value of 30 ± 20 MHz, in reasonable agreement with the predictions of other workers. No definite information is available for γ_{ab} for the 0·633 μm line, though Fork and Pollack (1965) have suggested a value of about 20 MHz. The results of the present work suggest a value of 30 ± 20 MHz. Finally it may be noted that the separation of the beat notes corresponding to $\nu_3 - \nu_2$ and $\nu_2 - \nu_1$ agree well with theory.

The Ar^+ system is rather different in that the linewidth appears to be of the order of the mode spacing (Bennett *et al.* 1966). It might be expected that the two- or three-mode approximation should not be valid since the degree of interaction between modes will be especially strong. This appears to be the case since the experiment results cannot be reconciled with any theoretical value of γ_{ab}.

6. Conclusions

The observations on the variation of beat frequency with varying cavity Q have been successfully described using Lamb's semiclassical theory for two and three modes. The results give information as to which mode distributions are stable and how such distributions vary with time due to small changes in the cavity. Values have been obtained for the constant γ_{ab} which are in reasonable agreement with those of other workers for the case of the He–Ne laser. The occurrence of straight-line graphs suggests, perhaps surprisingly, that the positions of the oscillating modes across the gain curve are fairly stable with time and are not continually changing. Finally, the theory appears not to work for the argon ion laser, which suggests that the Lamb three-mode theory may not be suitable for lasers in which the natural linewidth is large compared with the axial mode spacing.

Acknowledgments

The authors wish to thank Mr. P. Pollard for his invaluable technical assistance. One of us (M.D.S.) wishes to acknowledge the Science Research Council for providing a research studentship.

References

ALLEN, L., JONES, D. G. C., and SAYERS, M. D., 1968, *J. Sci. Instrum.*, **1**, 133–5.
ALLEN, L., JONES, D. G. C., and SIMANTIRIS, HELEN, 1967, *Proc. Phys. Soc.*, **91**, 774–8.
BENNETT, W. R., JR., 1962, *Phys. Rev.*, **126**, 580–93.
BENNETT, W. R., JR., BALLICK, E. A., and MERCER, G. N., 1966, *Phys. Rev. Lett.*, **16**, 603–5.
BENNETT, W. R., JR., KINDLMANN, P. J., and MERCER, G. N., 1965, *Appl. Opt. Suppl.*, **2**, 34–57.
BOLWIJN, P. T., 1965, *Phys. Lett.*, **19**, 384–5.
—— 1966, *J. Appl. Phys.*, **37**, 4487–92.
—— 1967, *Phys. Lett.*, **25**, A632–4.
FORK, R. L., and POLLACK, M. A., 1965, *Phys. Rev.*, **139**, A1408–14.
FOX, A. G., and LI, T., 1961, *Bell Syst. Tech. J.*, **XL**, 453–88.
JAVAN, A., BENNETT, W. R., JR., and HERRIOTT, D. R., 1961, *Phys. Rev. Lett.*, **6**, 106–10.
KISS, A., and SALAMON, T., 1967, *Rep. Cont. Res. Inst. Phys., Hungary*, **15**, 19–36.
LAMB, W. E., JR., 1964, *Phys. Rev.*, **134**, A1429–50.
MCFARLANE, R. A., 1964, *Phys. Rev.*, **3**, A543–50.
SCHLEUSENER, S. A., and READ, A. A., 1966, *Rev. Sci. Instrum.*, **37**, 287–9.
SCHWEITZER, W. G., JR., BIRKY, M. M., and WHITE, J. A., 1967, *J. Opt. Soc. Am.*, **57**, 1226–30.
WITTEMAN, W. J., 1966, *Philips Res. Rep.*, **21**, 73–84.